A Review of ABCD Parameters

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This article reviews transmission matrices (ABCD parameters), their definition, and their application in microwave circuit analysis. It presents their relationship to scattering parameters and demonstrates their use by analyzing a popular transistor amplifier circuit.

Several parameter sets exist for characterizing linear two-port networks. The most common parameter sets include admittance (y), impedance, (z), hybrid (h), and scattering (s) parameters. Although these sets can be used to describe a linear network with an arbitrary number of ports, this article will only discuss two-port networks. The terminal characteristics of a given two-port are represented by any of these parameter sets as a 2 × 2 matrix with each element generally being a complex number. These parameter representations can be used to describe both active and passive networks. It is important to note that these sets are valid only for linear networks. When these sets are used to model active devices the network must be operating under small signal conditions. Other parameter sets exist for describing large signal operation but they will not be discussed here.

It is helpful to understand that a given two-port can be represented by any of these parameter sets and that the terminal behavior of the network is completely described by that set. No new information is given in one set that is not available in another set's representation of the network. Formulas exist in many texts for conversion between the various parameter sets [1]. The differences lie in how the two-port sets are measured and how they are used. Certain circuit configurations are more easily characterized by one set than another. Parameter sets y, z, and h represent ratios of voltages and currents at the terminals of the two-port and are measured with open or short circuit port terminations. Hybrid parameters have commonly been used to describe transistor behavior at DC and low frequencies. Admittance parameters have been used extensively for transistor characterization at higher frequencies and many design methods exist for transistors based on this parameter set [2]. Series and parallel connections of two two-port networks are easily analyzed by the use of impedance and admittance parameters respectively.

Scattering parameters, mainly used to characterize networks at microwave frequencies, represent ratios of


$$V_1 = AV_2 + BI_2$$
$$I_1 = CV_2 + DI_2$$
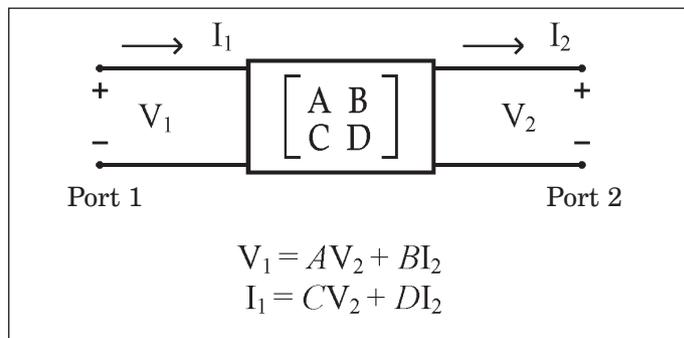
▲ **Figure 1. Two-port ABCD parameter definitions.**

|	
A = 1	B = Z
C = 0	D = 1
A = 1	B = 0
C = Y	D = 1
$A = \cos\beta l$	$B = jZ_0\sin\beta l$
$C = jY_0\sin\beta l$	$D = \cos\beta l$
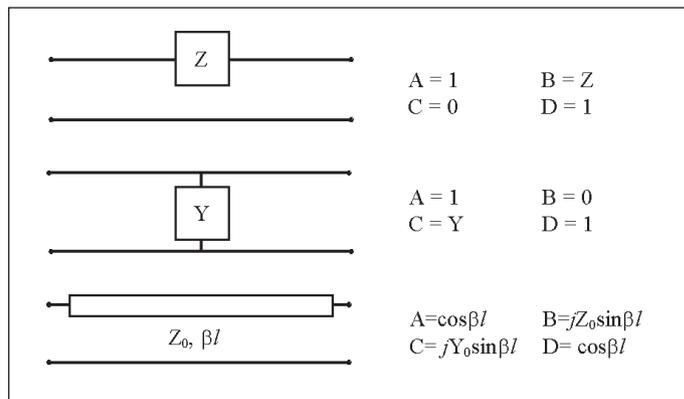
▲ **Figure 2. Some common two-port networks and their ABCD parameters.**

incident and reflected traveling waves (voltage or current). They are measured by terminating the network in a known real impedance (usually 50 ohms) rather than an open or short circuit. This is due to the difficulty in producing open and short circuits at high frequencies and because active devices can become unstable with highly reflective loads. Scattering parameters are also popular because powerful design and analysis methods have been developed using this parameter set.

Transmission or ABCD parameters are another 2 × 2 matrix parameter set that can be used to characterize linear two-port networks. The defining equations for an ABCD matrix in relation to a two-port are given in Figure 1. The individual parameters represent ratios of

the voltage or current at port 1 with the voltage or current at port 2. For example, $A = V_1/V_2$ with $I_2 = 0$ (open circuit at port 2), $B = V_1/I_2$ with $V_2 = 0$ (short circuit at port 2), and so on. It is evident that ABCD parameters are quite similar to hybrid parameters. The unique advantage of ABCD parameters is that a cascaded network of two or more two-ports can be easily analyzed by multiplying the individual transmission matrices. This can be very useful in microwave network analysis since many circuits consist of cascaded connections of both active and passive elements. Some common two-port networks along with their ABCD representations are shown in Figure 2. The following example will demonstrate the use of transmission matrices in circuit analysis.

A useful transistor amplifier circuit is the cascode buffer configuration. It consists of a common emitter transistor followed by a common base transistor as shown in Figure 3. This circuit has many advantages over a single transistor amplifier. The cascode configuration offers improved frequency response due to a reduction of the Miller effect. It has a much lower reverse transmission which provides higher isolation from output to input and generally provides higher gain. This circuit may find application between an antenna and a mixer in a receiver. The increased reverse isolation will reduce the amount of emissions at the antenna due to the local oscillator frequency. This circuit may also serve as a buffer following an oscillator to decrease the amount of load pulling although the small signal condition may not apply. A disadvantage of this circuit is the additional power supply voltage needed since V_{ceQ2} must be biased above V_{ceQ1}.

The transmission parameters for both Q1 and Q2 are needed in their respective configurations to analyze the circuit as a single two-port. This example uses the NE85633 operating at $I_c = 3$ mA, $V_{ce} = 2.5$ V and 900 MHz. Note that the two transistors do not need to have the same bias voltage but they must have the same collector current. The scattering parameters for this device in common emitter configuration at the above conditions (as provided by the manufacturer) are

$s_{11} = .495\angle-158°$ $s_{12} = .132\angle45°$
$s_{21} = 2.55\angle75°$ $s_{22} = .415\angle-52°$
$K = .922$ $|\Delta| = .188$

Again, the information contained in these parameters is sufficient to determine the ABCD representation of this network, eliminating any need to measure them directly (which may be difficult if not impossible). The appendix includes the relationships for converting between scattering and ABCD parameters. The ABCD parameters for Q1 can be derived directly from these relations while those for Q2 must be found by first converting the above data to common base configuration. This is straightforward using the relations in [2]
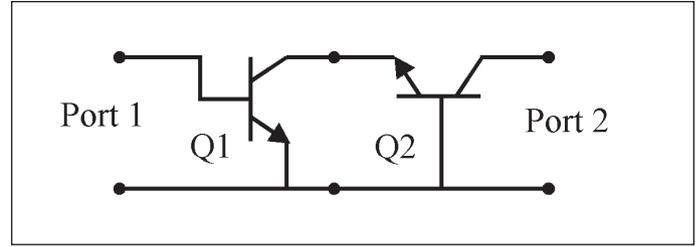
▲ **Figure 3. Circuit example used for illustration: a cascode transistor buffer amplifier.**

which further demonstrate that using any one parameter set in a given configuration completely describes the network for use in any other configuration. The transmission matrix for Q2 can then be found and is given in polar form along with that for Q1 as

$A_{Q1} = .086\angle-26°$ $A_{Q2} = .094\angle151°$
$B_{Q1} = 10.3\angle-116°$ $B_{Q2} = 11.2\angle61°$
$C_{Q1} = .005\angle-63°$ $C_{Q2} = .005\angle112°$
$D_{Q1} = .339\angle-73°$ $D_{Q2} = 1.01\angle18°$

Using matrix multiplication the equivalent two-port transmission matrix is found as

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_{Q1} & B_{Q1} \\ C_{Q1} & D_{Q1} \end{bmatrix} \begin{bmatrix} A_{Q2} & B_{Q2} \\ C_{Q2} & D_{Q2} \end{bmatrix}$$

Note that the order of the matrices on the right hand side is important since matrix multiplication is not commutative. The order of the matrices must appear in the same order as the circuit elements. The result, after converting back to scattering parameters, is found as

$s_{11} = .536\angle æ134°$ $s_{12} = .008\angle172°$
$s_{21} = 3.85\angle47°$ $s_{22} = 1.12\angle-30°$
$K = -3.59$ $|\Delta| = .572$

Note that the magnitude of the reverse transmission coefficient, s_{12}, has been significantly reduced compared to that of a single transistor. However, the network now suffers from greater instability as indicated by the factor K and the magnitude of s_{22}. In fact, a stability analysis shows that no passive source termination will make the reflection coefficient looking into the output less than one. This will restrict the value of the load termination needed for stability and may limit the application of this circuit as an amplifier. To demonstrate ABCD parameters further, the stability of this circuit will be improved by adding a shunt resistance at the output of the network. This resistive loading technique can sometimes be used to improve stability [3]. The resistance is in cascade with the network which makes ABCD parameters a useful method to analyze the loading effect as shown in
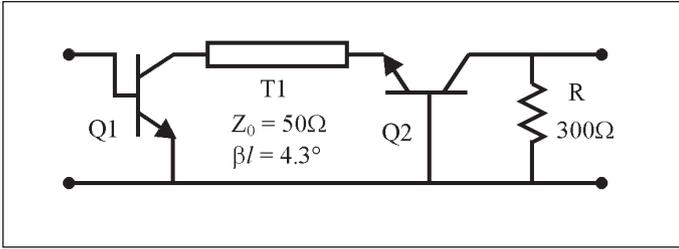
▲ **Figure 4. Resistive loading for improved stability.**

Figure 4. Also, a short transmission line is included between the two transistors for demonstration and for practicality in circuit realization. The values for the two added elements are also shown in the figure and their ABCD representations can be found from the relations in Figure 2. The resulting equation for the network is

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_{Q1} & B_{Q1} \\ C_{Q1} & D_{Q1} \end{bmatrix}\begin{bmatrix} A_{T1} & B_{T1} \\ C_{T1} & D_{T1} \end{bmatrix}\begin{bmatrix} A_{Q1} & B_{Q1} \\ C_{Q1} & D_{Q1} \end{bmatrix}\begin{bmatrix} A_R & B_R \\ C_R & D_R \end{bmatrix}$$

where the order of the matrices is once again emphasized. The resulting scattering parameters are

$$s_{11} = .553\angle{-134°} \qquad s_{12} = .007\angle{172°}$$
$$s_{21} = 3.38\angle{47°} \qquad s_{22} = .820\angle{-30°}$$
$$K = 4.57 \qquad |\Delta| = .434$$

The two-port maintains the high reverse isolation, the forward transmission is still high, but now the network is unconditionally stable. The transmission line is negligible but the 300 ohm resistor has greatly improved the usefulness of this circuit. The designer is now free to use whatever source and load terminations necessary to meet gain, VSWR, and noise requirements. For example, if maximum gain is needed the network may be designed with a simultaneous conjugate match. The transducer power gain for this network with this match is greater than 17 dB. Other terminations can be used to satisfy other requirements.

This analysis using ABCD parameters has proven quite simple and straightforward. The same analysis could have been performed with methods based on the original scattering parameters. For example, Mason's rule is a powerful approach that is well suited for cascaded as well as other connections of two-ports. However, this method becomes very cumbersome for networks containing more than a few elements. Transmission parameters are clearly easier to use for these types of networks.

This article has demonstrated the advantage of using ABCD parameters in analyzing cascaded connections of two-port networks. The simplified example above requires additional work to be done to account for stability at other frequencies and to account for noise, mismatch, and other performance criteria. Much of this additional work could also be done using transmission parameters. In fact, by combining ABCD parameters along with the other parameter sets, almost all types of linear networks can easily be analyzed. All these parameter representations serve as powerful tools in linear circuit analysis. ∎

References

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Author information

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$$A = \frac{(1+s_{11})(1-s_{22})+s_{12}s_{21}}{2\,s_{21}}$$

$$s_{11} = \frac{A + \dfrac{B}{Z_0} - CZ_0 - D}{\Delta T}$$

$$B = Z_0\,\frac{(1+s_{11})(1+s_{22})-s_{12}s_{21}}{2\,s_{21}}$$

$$s_{12} = \frac{2(AD - BC)}{\Delta T}$$

$$\text{where:} \quad \Delta T = A + \frac{B}{Z_0} + CZ_0 + D$$

$$C = \frac{1}{Z_0}\,\frac{(1-s_{11})(1-s_{22})-s_{12}s_{21}}{2\,s_{21}}$$

$$s_{21} = \frac{2}{\Delta T}$$

$$D = \frac{(1-s_{11})(1+s_{22})+s_{12}s_{21}}{2\,s_{21}}$$

$$s_{22} = \frac{-A + \dfrac{B}{Z_0} - CZ_0 + D}{\Delta T}$$

▲ **Appendix. *s* to ABCD parameter (left) and ABCD to *s* parameter (right) conversions.**