Lecture 15: Transformer Shunt Inductance. Tuned Transformers.

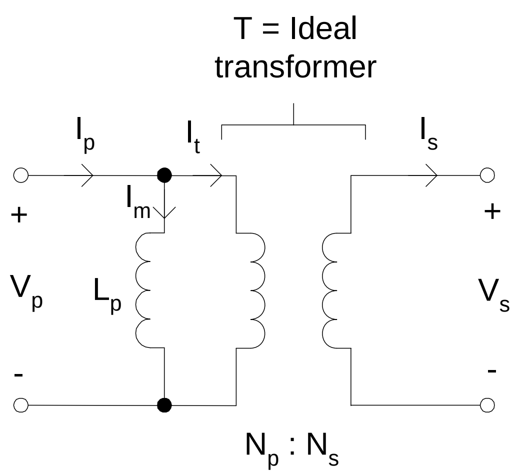
In the last lecture, we derived the transformer equations

$$V_p = \frac{N_p}{N_s} V_s \quad (6.21)$$

and

$$I_p = \underbrace{\frac{V_p}{j\omega L_p}}_{I_m} + \underbrace{\frac{N_s}{N_p} I_s}_{I_t} \quad (6.22)$$

where I_m = magnetization current and I_t = transformer current. An equivalent electrical circuit for such a nonideal transformer can be constructed from these two equations (Fig. 6.4a):



In particular, we have a shunt inductor that appears at the primary terminals of an *ideal* transformer.

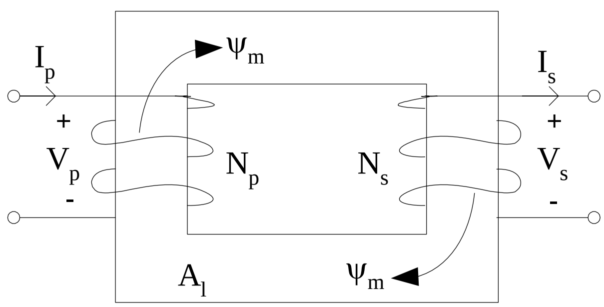
With the shunt inductance in the model, the high-pass nature of a physical transformer is properly accounted for, since at DC the primary terminals of *T* will be shorted.

Without the shunt L, the ideal transformer appears to transform voltages and currents equally well for all frequency, which cannot be true (by Faraday's Law).

Secondary Inductance

Equivalently, the shunt inductance can also have been incorporated from the secondary.

To do this, we begin again with Fig. 6.2:



and (6.12): $$\psi_m = N_p A_l I_p - N_s A_l I_s \quad (1)$$

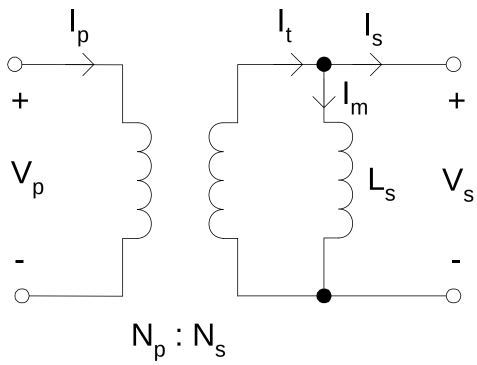
Solving for I_s $$I_s = \frac{N_p}{N_s} I_p - \frac{\psi_m}{N_s A_l} \quad (2)$$

Now, from (6.10) $V_s = N_s j\omega \psi_m$ or $$\psi_m = \frac{V_s}{N_s j\omega} \quad (3)$$

Substituting (3) into (2) leaves
$$I_s = \frac{N_p}{N_s} I_p - \frac{V_s}{j\omega \underbrace{N_s^2 A_l}_{L_s}} \quad (4)$$

or
$$I_s = \underbrace{\frac{N_p}{N_s} I_p}_{I_t} - \underbrace{\frac{V_s}{j\omega L_s}}_{I_m} \tag{5}$$

The equivalent electrical circuit for (6.21) and this last expression is (Fig. 6.4b):

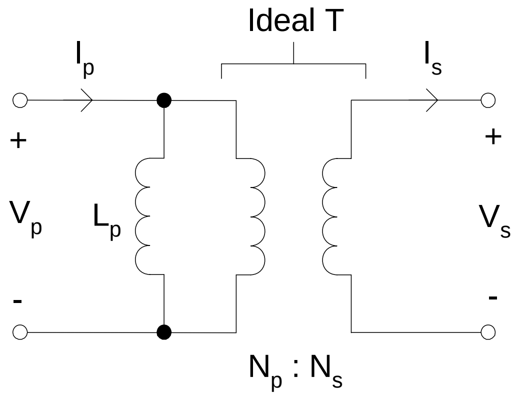


To check the directions for the current shown in this figure, we can apply KCL at the secondary:
$$I_t = I_m + I_s \qquad \text{or} \qquad I_s = I_t - I_m \tag{6}$$
The minus sign here agrees with (5).

To summarize this work so far, whether the magnetization current effect is included on the primary side or the secondary side of the transformer is immaterial: they are equivalent.

Actually, we can develop this latter secondary inductance equivalent circuit simply from the impedance transformation property of the ideal transformer!

Begin with



We've seen previously that for an **ideal transformer**

$$Z_s = \left(\frac{N_s}{N_p}\right)^2 Z_p \qquad (7), (6.19)$$

Here $$Z_p = j\omega L_p \quad \text{and} \quad L_p = A_l N_p^2 \qquad (8), (6.23)$$

so that $$Z_s = \frac{N_s^2}{N_p^2} j\omega\left(A_l N_p^2\right) = j\omega \underbrace{A_l N_s^2}_{L_s}$$

or $$Z_s = j\omega L_s$$

which we can model simply as an ideal transformer with a shunt L_s as shown on the previous page (Fig. 6.4b).

Tuned Transformers

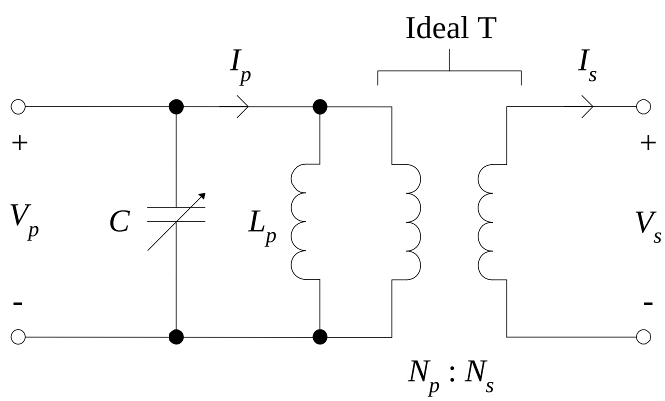
Achieving impedance match between the various subsystems in a multistage communications circuit is very important. Otherwise, precious signal is needlessly wasted.

Transformers – specifically *ideal* transformers – can be used as matching networks since, as we've already seen,

$$Z_p = \left(\frac{N_p}{N_s}\right)^2 Z_s \quad \Omega \tag{6.19}$$

We can choose N_p/N_s to change (or "transform") Z_p to a desired value for matching.

Note that (6.19) is valid only for *ideal* transformers. One way to negate the effects of the magnetization current I_m in a practical transformer (so that the ideal T equations apply) is to use a **tuning capacitor**:



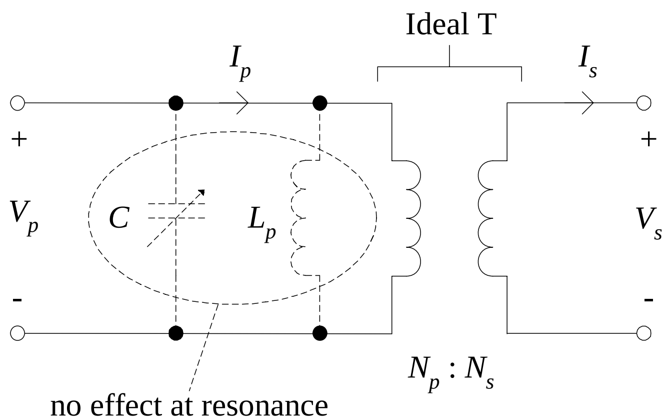
We can adjust C to **resonate out** the effects of L_p at the desired frequency of operation. That is, suppose the transformer is designed to operate at $f = f_0$. For an LC resonance at $f_0 = 1/\left(2\pi\sqrt{LC}\right)$, then adjust C such that

$$C = \frac{1}{4\pi^2 L f_0^2} \quad \text{F} \tag{9}$$

Consequently, now at this operating frequency f_0

$$Z_c \,||\, Z_{L_p} = \frac{Z_c Z_{L_p}}{Z_c + Z_{L_p}} = \infty$$

and the equivalent circuit for this tuned transformer circuit becomes



which is simply an ideal transformer. Very cool!

This resonant method is only a narrow-band solution, but it can be extremely useful. Capacitive transformer tuning effectively makes a band-pass filter from a high-pass filter.

Examples

The two tuned transformers in the NorCal 40A are T2 (RF Filter) and T3 (matching between RF Mixer and IF Filter). Let's consider both of these quickly once again in the light of our expanded understanding of transformers.

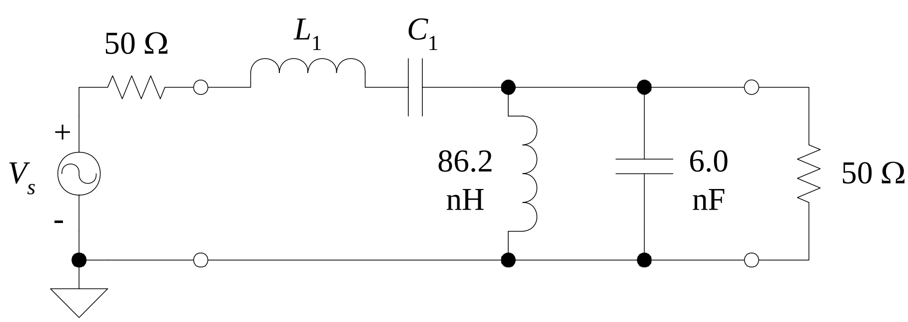
1. T3 (between RF Mixer and IF Filter): This transformer was also considered in the previous lecture. T3 is used to transform the output impedance from the RF Mixer to match the input impedance of the IF Filter (200 Ω).

From the data sheet for the SA602AN IC, the output impedance is $2\times 1500\ \Omega = 3000\ \Omega$. Using (7):

$$Z_s = \left(\frac{N_s}{N_p}\right)^2 Z_p = \left(\frac{6}{23}\right)^2 3000 = 204.2\ \Omega$$

which is very close to the desired 200 Ω for the IF Filter!

2. T2 (RF Filter): Consider once again the second order Butterworth bandpass filter example we discussed earlier in Lecture 12:



L1 and C1 are components that are soldered onto your PCB. Where do the 86.2 nH and 6.0 nF components come from? As we mentioned earlier in Lecture 12, they both come from T2!

To see this explicitly for T2, $L_p = A_l N_p^2 = 66\times 10^{-9}$ H/turn$^2 \cdot 1^2$ = 66 nH, which is close to the 86.2 nH shown above that is needed for a second-order bandpass filter.

What about the C? That comes from C2 and the impedance transforming properties of T2:

$$Z_p = \left(\frac{N_p}{N_s}\right)^2 Z_s \quad \Rightarrow \quad C_p = \left(\frac{N_s}{N_p}\right)^2 C_s$$

With $N_s = 20$, $N_p = 1$ and $C_s = 15$ pF, then

$$C_p = \left(\frac{20}{1}\right)^2 15 \text{ pF} = 6 \text{ nF}$$

which is exactly the value needed for the second order Butterworth bandpass filter! Very cool.