# Lecture 6: Parallel Resonance and Quality Factor. Transmit Filter.

As we saw in the last lecture, in order for a series RLC circuit to possess a large $Q$ the reactance of $L$ or $C$ (at resonance) must be much larger than the resistance:

$$Q_s = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R} \qquad (3.90)$$

Consequently, if we desire a large $Q$ (very good frequency selectivity) in a series "tank" circuit, the resistance should be relatively small (for a reasonable $X$).

For example, a resistance of 50 Ω - 75 Ω is common for receivers and some antennas. This relatively small resistance is seen by the RF Filter in the NorCal 40A, which uses a series RLC filter.

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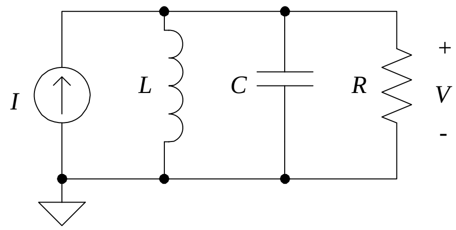
## Parallel Resonance

However, if the "load" resistance in the circuit is relatively *large*, it becomes more difficult to achieve the high reactances at resonance necessary for a high-$Q$ series RLC circuit.

If this is the case – and it often is in the NorCal 40A – then a designer needs to use a parallel resonant RLC circuit (Fig. 3.7):

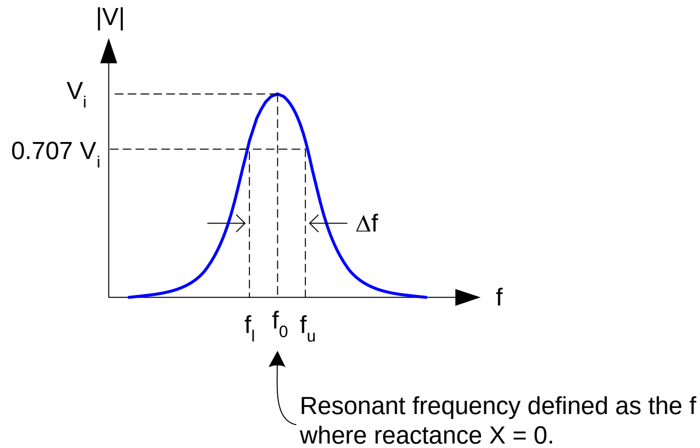



For this parallel RLC circuit

$$Q_p = \frac{\text{resistance}}{\text{reactance}\big|_{\omega_0}}$$

which is the inverse of $Q_s$.

Following the analysis in the text (Section 3.7), we find that the frequency response is similar to a series RLC circuit




However,
$$Q_p = \frac{R}{\omega_0 L} = \omega_0 CR \qquad (3.109)$$

For a high-$Q$ parallel resonant circuit, we need a small reactance of $L$ or $C$ (at resonance) compared to the resistance. Sometimes this is easier to do when $R$ is large in a parallel RLC circuit than with a high reactance in a series RLC circuit.

# **Examples**:

7.00E+06 = resonant frequency

15 = $Q_p = Q_s = Q$

| R (Ω) | Series RLC | | Parallel RLC | | Check: | |
|---|---|---|---|---|---|---|
| | $L_s=RQ/\omega_0$ | $C_s=1/(\omega_0 RQ)$ | $L_p=R/(\omega_0 Q)$ | $C_p=Q/(\omega_0 R)$ | $f_{0,s}=1/(2\pi Sqrt[L_sC_s])$ | $f_{0,p}=1/(2\pi Sqrt[L_pC_p])$ |
| 50 | 1.71E-05 | 3.03E-11 | 7.58E-08 | 6.82E-09 | 7.00E+06 | 7.00E+06 |
| 500 | 1.71E-04 | 3.03E-12 | 7.58E-07 | 6.82E-10 | 7.00E+06 | 7.00E+06 |
| 1500 | 5.12E-04 | 1.01E-12 | 2.27E-06 | 2.27E-10 | 7.00E+06 | 7.00E+06 |
| 3000 | 1.02E-03 | 5.05E-13 | 4.55E-06 | 1.14E-10 | 7.00E+06 | 7.00E+06 |

The green highlighted case is close to the RF Filter in the NorCal 40A (series RLC).

7.00E+06 = resonant frequency

100 = $Q_p = Q_s = Q$

| R (Ω) | Series RLC | | Parallel RLC | | Check: | |
|---|---|---|---|---|---|---|
| | $L_s=RQ/\omega_0$ | $C_s=1/(\omega_0 RQ)$ | $L_p=R/(\omega_0 Q)$ | $C_p=Q/(\omega_0 R)$ | $f_{0,s}=1/(2\pi Sqrt[L_sC_s])$ | $f_{0,p}=1/(2\pi Sqrt[L_pC_p])$ |
| 50 | 1.14E-04 | 4.55E-12 | 1.14E-08 | 4.55E-08 | 7.00E+06 | 7.00E+06 |
| 500 | 1.14E-03 | 4.55E-13 | 1.14E-07 | 4.55E-09 | 7.00E+06 | 7.00E+06 |
| 1500 | 3.41E-03 | 1.52E-13 | 3.41E-07 | 1.52E-09 | 7.00E+06 | 7.00E+06 |
| 3000 | 6.82E-03 | 7.58E-14 | 6.82E-07 | 7.58E-10 | 7.00E+06 | 7.00E+06 |

Three things we can observe from these examples:

1. For higher $Q$ in a series RLC (at fixed $R$) we need to use a larger $L$ or a smaller $C$ (larger reactance for $L$ and $C$).
2. For higher $Q$ in a parallel RLC (at fixed $R$) we need to use a smaller $L$ or a larger $C$ (smaller reactance for $L$ and $C$).
3. For a fixed $R$, smaller $L$ and larger $C$ (smaller reactance for both $L$ and $C$) are needed in a parallel RLC circuit to achieve the same $Q$ as a series RLC.

Also, as mentioned in the text,

$$Q_p = \frac{f_0}{\Delta f} \tag{3.110}$$

which is the same expression as for $Q_s$.

A number of parallel RLC circuits are used in the NorCal 40A. One of these is the Transmit Filter constructed in Prob. 9.

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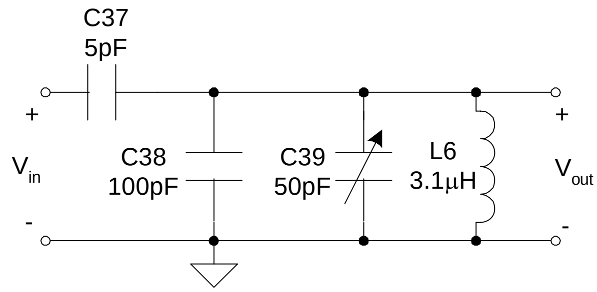
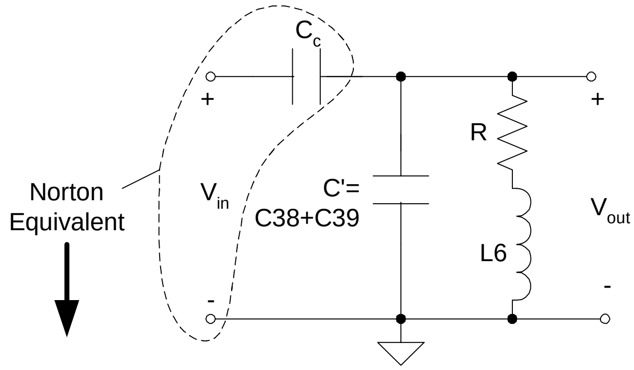
## Transmit Filter

The Transmit Filter is a “modified parallel RLC circuit” (see inside front cover). The Transmit Filter is mainly used to filter all harmonics other than 7 MHz coming from the Transmit Mixer (see Fig. 1.13):




This is not a true parallel RLC circuit in the sense of Fig. 3.7a.

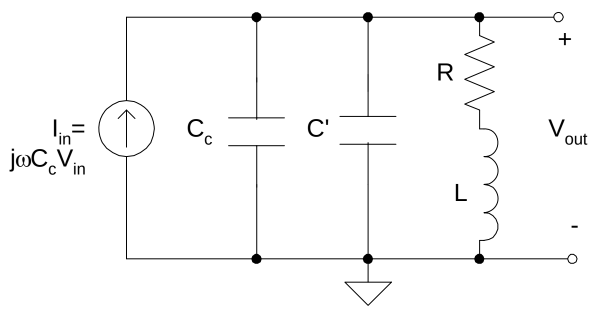
It is important to analyze this circuit for use in Prob. 9. First, we’ll combine C38 and C39 and include losses from L6:



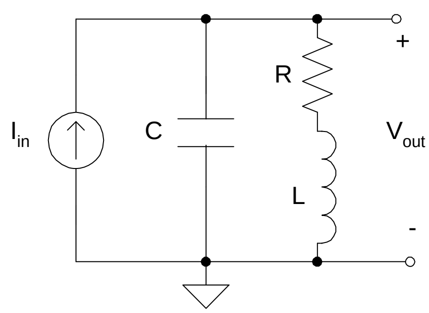

We'll use a Norton equivalent circuit for $V_{\text{in}}$ and $C_c$:

$$I_{\text{sc}} = \frac{V_{\text{in}}}{\left(j\omega C_c\right)^{-1}} = j\omega C_c V_{\text{in}} \quad \text{and} \quad Z_{\text{Th}} = \frac{1}{j\omega C_c}$$

Using this in the previous circuit:




which we can reduce to

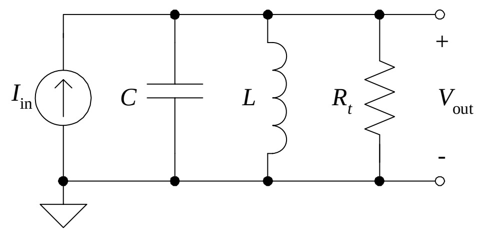



This is still not a parallel RLC form. Instead, this is called an "RL-parallel-C" (RL||C) circuit. The characteristics of this circuit (and the RC||L) are listed at the end of this lecture (from Krauss, *et al*. “Solid State Radio Engineering”).

Here are those characteristics of the RL||C circuit important to us right now:

| **Quantity** | **Exact Expression** | **Approximate Expression ($Q_t$>10)** |
|---|---|---|
| $\omega_0$ | $= \left( \frac{1}{LC} - \frac{R^2}{L^2} \right)^{1/2}$ | $\approx \frac{1}{\sqrt{LC}}$ |
| $Q_t$ | $= \frac{\omega_0 L}{R} = \omega_0 C R_t$ | $\approx \frac{1}{\omega_0 C R}$ |
| $R_t$ | $= \frac{L}{CR} = \frac{Q_t}{\omega_0 C} = R\left(Q_t^2 + 1\right)$ | $\approx Q_t^2 R = \omega_0 L Q_t$ |

What is particularly relevant here are the approximate expressions for large $Q_t$. These are precisely the same equations for a standard RLC circuit, but with an $R_t$:




$R_t$ is not an actual resistor but rather an effective resistance. This $R_t$ is quite large in the Transmit Filter. Let’s check some numbers for the NorCal 40A:

✓ $C = C_c + \text{C38} + \text{C39} \approx 5 + 100 + 38 = 143 \text{ pF}$,

✓ $L = \text{L6} = 3.55 \ \mu\text{H}$,

✓ $R_t \approx Q_t^2 R \approx \left(\dfrac{\omega_0 L}{R}\right)^2 R = \dfrac{(\omega_0 L)^2}{R} = \dfrac{(\omega_0 \text{L6})^2}{R_{\text{L6}}}$

What is the resistance of L6 at 7 MHz? Not as obvious as it looks. (Due to the skin effect, the resistance of wire changes with frequency.) Let's <u>assume</u> $R_{\text{L6}} \approx 1 \ \Omega$. Then,

$$R_t \approx \frac{(\omega_0 \text{L6})^2}{R_{\text{L6}}} = (2\pi \cdot 7 \cdot 3.1)^2 = 18.6 \text{ k}\Omega$$

Whoa! That's big. The $Q$ of this RL||C Transmit Filter should then be

$$Q_t \approx \sqrt{\frac{R_t}{R}} = \sqrt{18{,}600} \approx 136.$$

That's a respectable $Q$ for a discrete-element RLC circuit. Your measured value will probably be less than this (more losses).

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## Summary

The Transmit Filter shown on p. 4 of this lecture can be modeled by an *effective* parallel RLC circuit shown on the previous page.

It is emphasized that $R_t$ is not an actual resistor, but rather an effective resistance due to losses in L6 and other effects mentioned in Prob. 9. You can use the analysis shown here to help with your solution and measurements in Prob. 9.

Lastly, you will find through measurements that this "modified parallel RLC circuit" has a much larger *Q* than if C37 were removed (yielding just a regular parallel RLC tank). Interesting!

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## Winding Inductors and Soldering Magnet Wire

You will wind the inductor L6 that is used in the Transmit Filter. It is specified in the circuit schematic to be constructed from 28 turns of wire on a T37-2 core, which is a toroid of 0.37-in diameter constructed from a #2-mix iron powder.

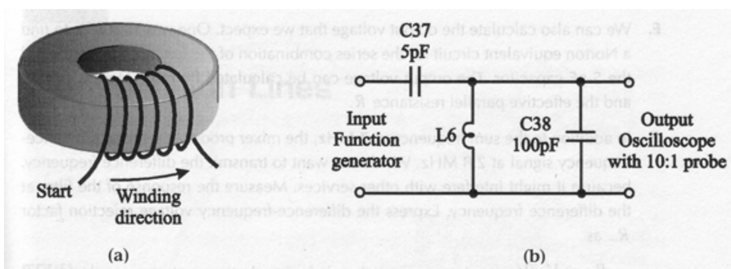


Figure 3.10. (a) Winding a toroid. Six turns are shown. (b) Initial band-pass filter connections.

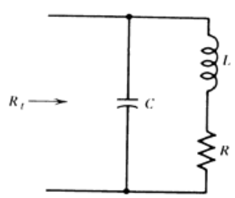
It is important you develop the habit of winding the toroids as illustrated in the text and in the NorCal40A *Assembly Manual* tutorial. Hold the toroid in your left hand and begin threading the wire through the top center of the toroid with your right. Each time the wire passes through the center of the toroid is considered one turn.

You'll need a small piece of fine sandpaper to clean the varnish off the ends of the magnet wire before soldering. Not removing **ALL** of the varnish will likely cause big problems because of poor ohmic contact. I check the continuity between the solder pad and a portion of the magnet wire sticking up through the solder meniscus to confirm good ohmic contact.

**Appendix 3-1**

# Tables of Design Formulas

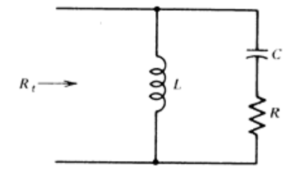
**TABLE 3-3.1 Design Formulas for the Resonant *RL*||*C* Circuit**




| Quantity | Exact Expression | Units | Approximate Expression, $Q_t \geq 10$ |
|---|---|---|---|
| $\omega_o$ | $= \left(\frac{1}{LC} - \frac{R^2}{L^2}\right)^{1/2}$ | rad/s | $\approx \frac{1}{\sqrt{LC}}$ |
| $Q_t$ | $\equiv \frac{\omega_o L}{R} = \omega_o C R_t$ | | $\approx \frac{1}{\omega_o CR}$ |
| $\omega_o L$ | $= \frac{1}{\omega_o C}\left(\frac{Q_t^2}{Q_t^2+1}\right)$ | ohms | $\approx \frac{1}{\omega_o C}$ |
| $R_t$ | $= \frac{L}{CR} = \frac{Q_t}{\omega_o C}$ <br> $= R(Q_t^2+1)$ | ohms | $\approx Q_t^2 R = \omega_o L Q_t$ |
| $B$ | | hertz | $\approx \frac{1}{2\pi C R_t} = \frac{R}{2\pi L} = \frac{f_o}{Q_t}$ |



**TABLE 3-3.2 Design Formulas for the Resonant *RC*||*L* Circuit**




| Quantity | Exact Expression | Units | Approximate Expression $Q_t \geq 10$ |
|---|---|---|---|
| $\omega_o$ | $= \left(\frac{1}{LC - R^2C^2}\right)^{1/2}$ | rad/s | $\approx \frac{1}{\sqrt{LC}}$ |
| $Q_t$ | $\equiv \frac{1}{\omega_o CR} = \frac{R_t}{\omega_o L}$ | | $\approx \frac{\omega_o L}{R}$ |
| $\omega_o L$ | $= \frac{1}{\omega_o C}\left(\frac{Q_t^2+1}{Q_t^2}\right)$ | ohms | $\approx \frac{1}{\omega_o C}$ |
| $R_t$ | $= \frac{L}{CR} = \omega_o L Q_t$ <br> $= R(Q_t^2+1)$ | ohms | $\approx Q_t^2 R = \frac{Q_t}{\omega_o C}$ |
| $B$ | | hertz | $\approx \frac{f_o}{Q_t} = \frac{1}{2\pi C R_t}$ |



(From H. L. Krauss, C. W. Bostian and F. H. Raab, *Solid State Radio Engineering*. New York: John Wiley & Sons, 1980.)