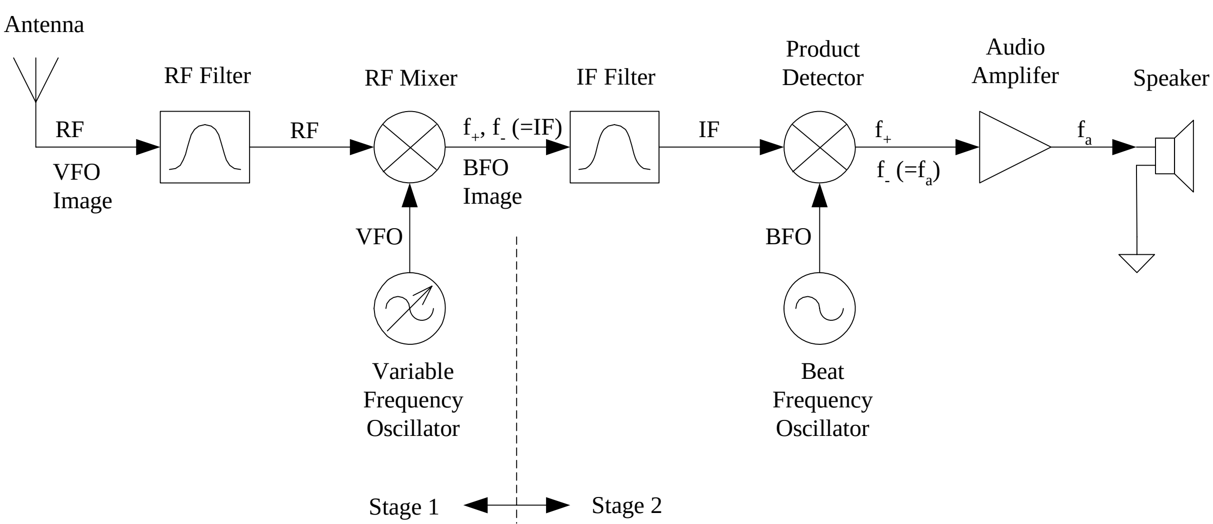
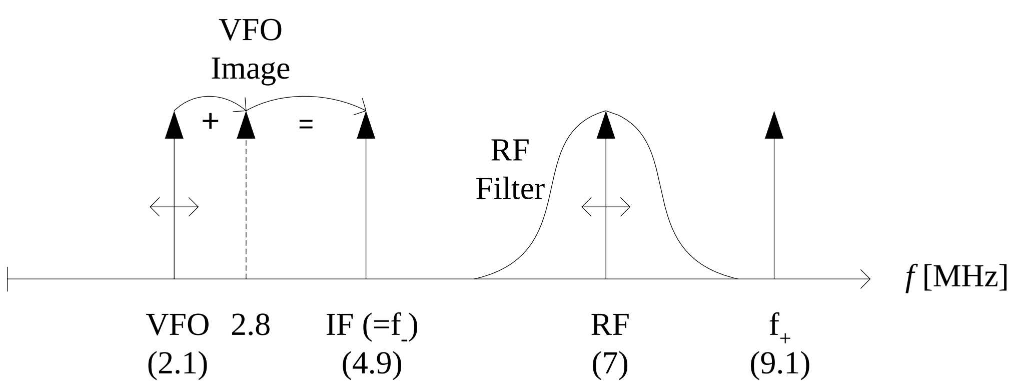
Lecture 13: Impedance Inverter. Cohn Crystal Filter.

A block diagram of a superhet receiver is shown below. Recall in the superhet receiver that the RF signal is mixed with the VFO signal by the RF Mixer down to the IF. In the NorCal 40A, the IF is approximately 4.9 MHz.

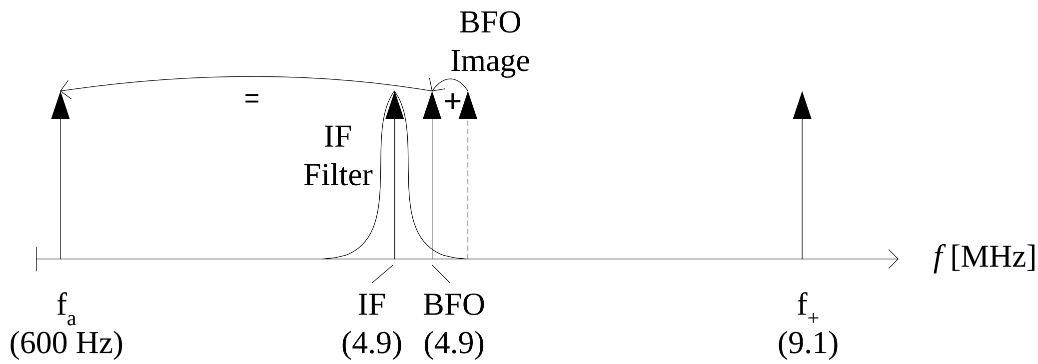


Stage 1:



After this, the IF signal is mixed with the BFO signal by the Product Detector down to audio frequencies (approximately 620 Hz in the NorCal 40A).

Stage 2:



One difficulty here is that the BFO image is only about 1.2 kHz away from the IF. For a center frequency of 4.9 MHz, we need a bandpass IF filter with a Q of approximately

$$Q_{\substack{\text{IF}\\\text{filter}}} = \frac{f_0}{\Delta f} \gtrapprox \frac{4.9\times 10^6}{500} \approx 10{,}000$$

That's a large Q! The Q's we've seen for discrete elements (and TL resonators) have been approximately 200 or less. That's 50x too small.

Quartz crystals will be used instead to achieve this high Q.

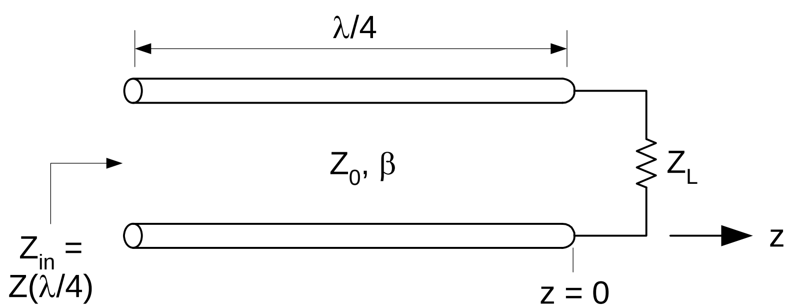
However, we need **both** series and parallel resonant elements to realize bandpass ladder filters. While crystals have both these resonances, they occur at different frequencies. We need these resonant frequencies to be the same.

So, how do we make a bandpass filter with identical quartz crystals? We will couple them together in a special way using impedance inverters.

Impedance Inverters

An impedance inverter is a device or circuit that has an input impedance inversely proportional to the load impedance. More specifically, the normalized input impedance equals the normalized load admittance.

Actually, we've already seen an example of an impedance inverter already: a $\lambda/4$ length of transmission line.



Recall from Lecture 9 that for an open circuit load ($Z_L = \infty$), $V(\lambda/4) = 0$ and $I(\lambda/4) = $ maximum. Therefore, $Z(\lambda/4) = 0$. Consequently, this TL has "inverted" the load impedance.

This is a general fact true of any load impedance connected to a $\lambda/4$ length of TL. As shown in the text (Section 4.11)

$$\frac{Z(\lambda/4)}{Z_0} = \frac{Z_0}{Z(0)} \qquad (4.102)$$

We make the following definitions.

- $z(\lambda/4) = Z(\lambda/4)/Z_0$ the normalized input impedance,

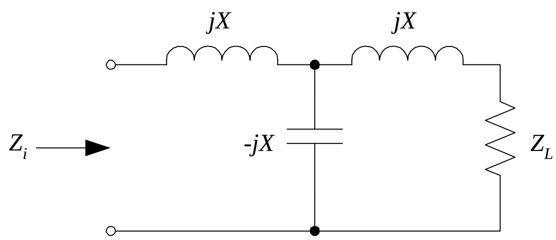
- $Z_0 / Z(0) = 1/z_L = y_L$ the normalized load admittance.

Then (4.102) can be written in the compact form

$$z\left(\frac{\lambda}{4}\right) = \frac{1}{z_L} \qquad (4.105)$$

We see here that the normalized input impedance equals the inverse of the normalized load impedance. This is the definition of an impedance inverter device.

Such a TL impedance inverter would be impractically long for our uses. Instead, we can make an impedance inverter using discrete *L*'s and *C*'s. From Fig. 5.14:



Notice that the magnitudes of the inductive and capacitive reactances are equal in this circuit. This strictly can occur only at a single frequency. So this impedance inverter will be a narrow-band device, which is ok for us since the IF Filter will have a very narrow passband.

Let's verify the operation of the impedance inverter in Fig. 5.14. The input impedance is

$$Z_i = jX + (jX + Z_L) \| (-jX)$$

$$= jX + \frac{(jX + Z_L)(-jX)}{\cancel{jX} + Z_L - \cancel{jX}} = jX + \frac{X^2 - jXZ_L}{Z_L} = \frac{X^2}{Z_L}$$

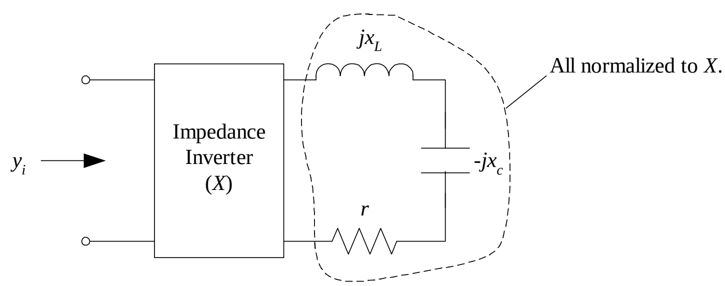
Therefore
$$\frac{Z_i}{X} = \frac{X}{Z_L} \quad \text{or} \quad z_i = \frac{1}{z_L} \tag{5.40}$$

where z_i and z_L are the normalized input and load impedances, respectively.

That is, the normalized input impedance equals the inverse of the normalized load impedance. All quantities have been normalized to the inverter reactance, *X*.

Cascade An Impedance Inverter to a Series Resonator

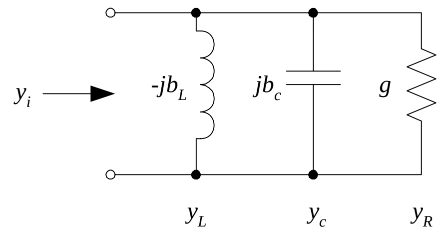
Now, let's examine what happens when an impedance inverter is placed in front of a series resonant circuit (Fig. 5.15a):



The impedance inverter, according to (5.41), provides a normalized input admittance y_i of

$$y_i = z_L = jx_L - jx_c + r \tag{5.42}$$

What does the RHS of (5.42) represent for y_i? It is a parallel resonant circuit! To see this, consider:



By inspection we see that

$$y_i = -jb_L + jb_c + g \tag{5.43}$$

Consequently, this circuit is equivalent to a series RLC with an impedance inverter provided [comparing (5.42) and (5.43)]

$$jx_L = jb_c,\ -jx_c = -jb_L \text{ and } r = g$$

or

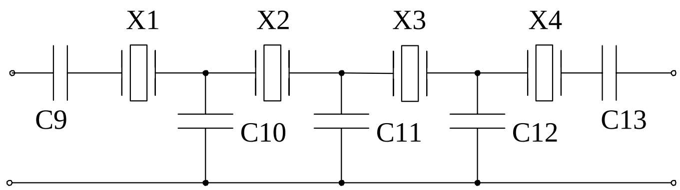
$$b_c = x_L,\ b_L = x_c \text{ and } g = r \qquad (5.44, 45, 46)$$

Hence, we conclude that a series RLC circuit connected through an impedance inverter appears to the input terminals of the inverter exactly equivalent to a parallel RLC circuit. Cool!

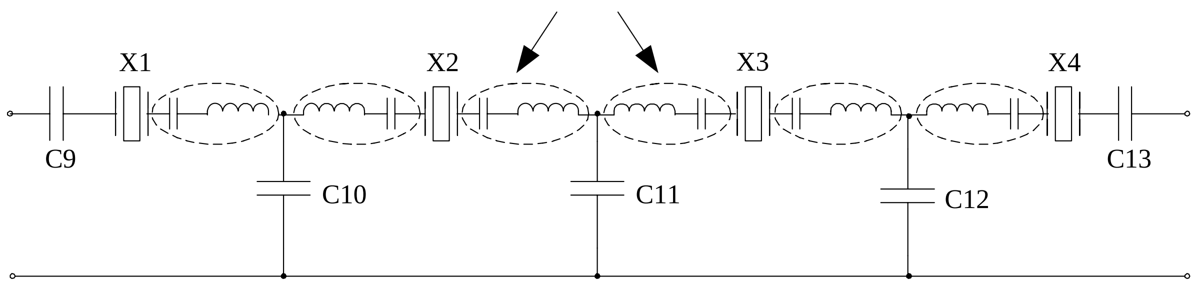
Cohn Filter

Your text provides a wonderful description of how the IF Filter works in the NorCal 40A. This filter is a fourth-order Cohn filter built from four quartz crystals and five identical capacitors.

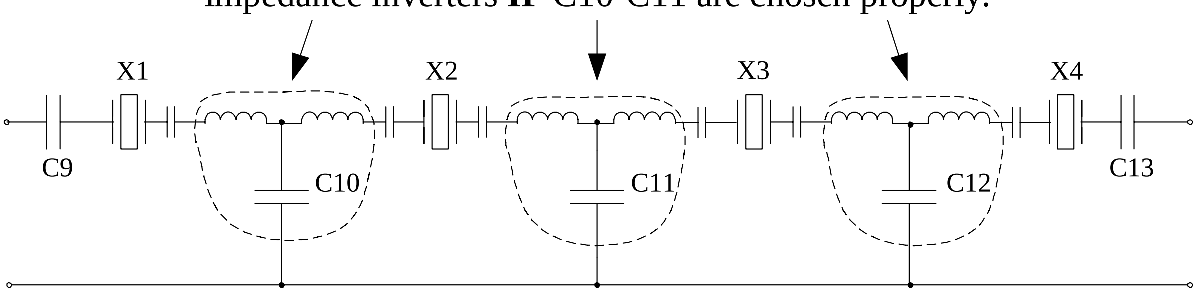
The four element Cohn filter that forms the Intermediate Frequency (IF) Filter in the NorCal 40A is:



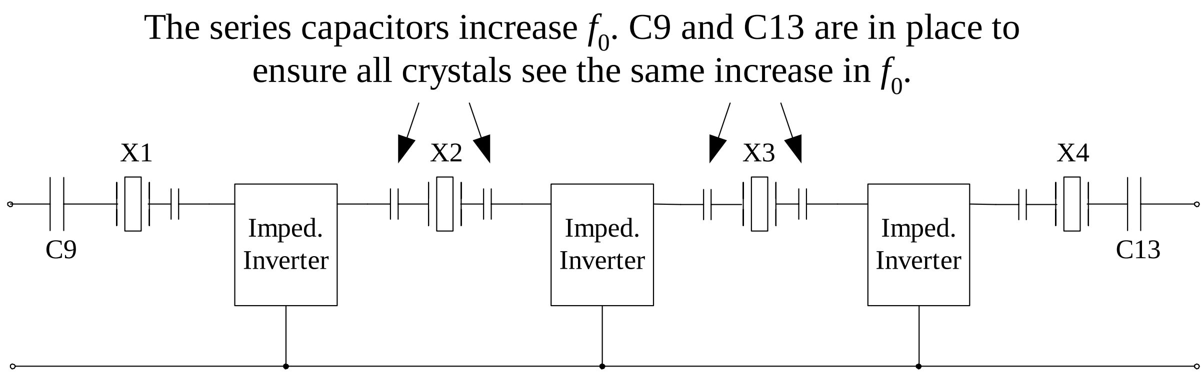
To understand the operation of this filter, the text first adds fictitious *L* and *C* elements. The unsigned reactances of *L* and *C* are equal so their series impedance is zero at f_0.



The next step is to recognize the presence of impedance inverters positioned between each quartz crystal.

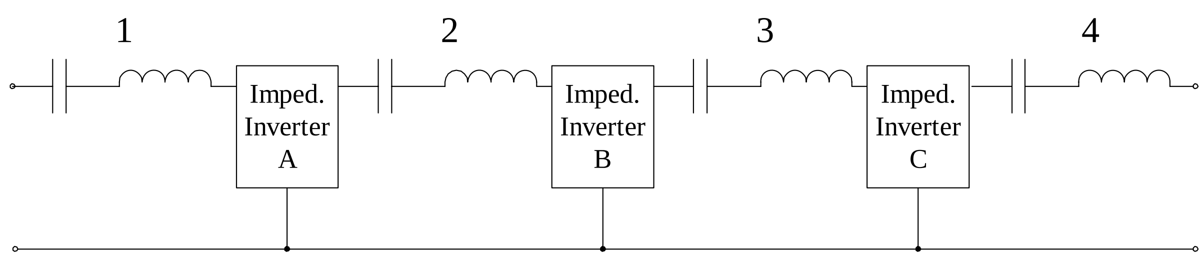


Replace these tee networks with impedance inverter circuits:



What remains are two capacitors in series with the crystals X2 and X3. We can now see that the purpose for C9 and C13 is to ensure that X1 and X4 also see the same capacitances.

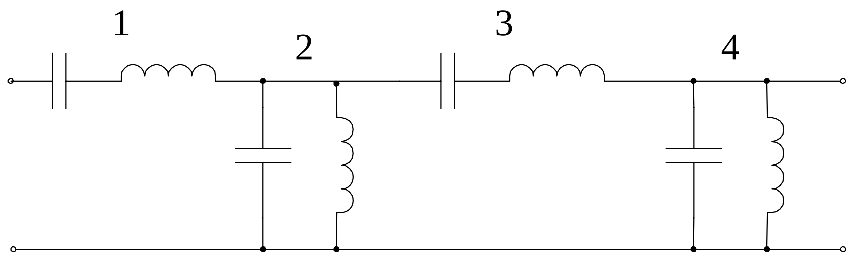
Now, substitute the equivalent series LC network for each of the quartz crystals (and the two series *C*'s):



Our qualitative analysis will begin at the far right of this equivalent circuit:

1. The series LC network 4 connected to impedance inverter *C* appears as a parallel LC network at the left. This is connected to series LC network 3.
2. The series network 3 – and now "parallel" network 4 – appears through impedance inverter B as "parallel" network 3 and series 4.

3. Finally, the series network 2, "parallel" 3 and series 4 appear to 1 through impedance inverter A as series 1 connected to "parallel" 2, series 3 and "parallel" 4, as shown below.



We can recognize this equivalent circuit as a fourth-order, bandpass LC ladder filter! Consequently, this fourth-order Cohn filter using quartz crystals is *effectively* a fourth-order, bandpass LC ladder filter.

Lastly, as mentioned earlier, C9-C13 must be chosen properly if they are to facilitate the impedance inverter operation. In particular, their reactance at 4.91 MHz (the IF) must closely match *L* (which includes the *L* of the crystal and the "fictitious" *L* of the impedance inverter).

While not quantitative in nature, the discussion here at least illustrates how the Cohn filter achieves its bandpass nature.

Loaded and Unloaded Q

In Prob. 14, you will measure the loaded Q of the crystal defined as

$$Q_{\text{loaded}} = \frac{X\big|_{\omega_0}}{R_m + R_{ckt}}$$

In other words, stating that a Q value is a "loaded Q" implies that losses from the crystal and the circuit it's connected to are **both** included. You'll likely measure $Q_{\text{loaded}} \approx \mathcal{O}(10{,}000)$.

The unloaded Q of the quartz crystal

$$Q_{\text{crystal}} = \frac{X\big|_{\omega_0}}{R_m}$$

is typically much larger since it includes only the losses ***in*** the crystal. As we've mentioned before, $Q_{\text{crystal}} \approx \mathcal{O}(150{,}000)$.