Avoiding RF Oscillation

By recognizing potential RF instability, you can protect yourself from unpleasant surprises. This article reviews RF circuit stability concepts and offers practical ways to guarantee stable operation.

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At low frequencies the Nyquist criteria provides a safe indication of stability. As frequencies increase circuit and system designers face a more difficult and tedious task. A thorough stability analysis should be performed through a wide range of frequencies, possible terminations, and power levels. Since true broadband nonlinear models are generally not available for the active devices, stability is evaluated at individual frequencies, based on small-signal device parameters.

A common mistake is to examine only the passband of the system, unfortunately not always sufficient. When out-of-band instability, particularly at low frequencies, is neglected it may show up in unwanted oscillation. Then, the first incoming signal, even noise, can turn an intended amplifier into a comb generator.

While a complete nonlinear stability analysis is more than we can cover here, the following discussion, covering linear small-signal circuit applications, also can be used as a start for large-signal analysis.
Possible causes of oscillation

Before getting specific, let us recognize that by using the appropriate feedback, an active twoport always can be turned into an oscillator at frequencies up to \( f_{\text{max}} \), the frequency at which the matched unilateral gain of the twoport is unity. Feedback may have been included by the engineer in the original design scheme or it may be completely unintentional. Oscillator designers use positive feedback deliberately, but an unwary amplifier designer may achieve it and find that his amplifier is actually an oscillator, due possibly to such feedback effects as poor grounding.

Since there is no doubt that feedback can cause instability, this article focuses on finding out if a twoport may oscillate with an arbitrary set of source and load terminations (Figure 1). In the following discussion, we’ll assume that our terminations have positive real parts. That is, the terminations do not have reflection coefficients whose magnitudes exceed one.

![Figure 1. An active twoport, characterized by its scattering matrix \( S \), may oscillate if either of the loop-gain products \( \Gamma_L \) or \( \Gamma_S \) exceeds unity. If the source and load terminations \( \Gamma^\text{in} \) and \( \Gamma^\text{out} \) are passive, then \( |\Gamma^\text{in}| \) or \( |\Gamma^\text{out}| \) must be greater than unity to satisfy the minimum loop-gain requirement.](image)

An early form of twoport stability was defined by Linvill[1] as the ability to conjugate match a twoport simultaneously with positive real terminations, without the possibility of oscillation. Later, it was shown that simultaneous conjugate-match is not always the most severe test of stability. An active twoport must be tested for all possible source and load terminations of positive real parts, to see if one of the reflection coefficients, \( \Gamma^\text{in} \) or \( \Gamma^\text{out} \), of the two port may have magnitudes greater than 1.0.[2]

**Analytic definition of twoport RF stability — the K-factor**

Mathematically, unconditional twoport stability exists when:

\[
\left| \Gamma_S \Gamma^\text{IN} \right| < 1
\]

and

\[
\left| \Gamma^\text{OUT} \Gamma_L \right| < 1
\]

for all

\[
\left| \Gamma_S \right| \leq 1
\]

\[
\left| \Gamma_L \right| \leq 1
\]

which implies that

\[
\left| \Gamma^\text{IN} \right| = \left| s_{11} + \frac{s_{12}s_{21}\Gamma_L}{1-s_{22}\Gamma_L} \right| < 1
\]

and

\[
\left| \Gamma^\text{OUT} \right| = \left| s_{22} + \frac{s_{12}s_{21}\Gamma_L}{1-s_{11}\Gamma_S} \right| < 1
\]

The above is guaranteed when the stability (K)-factor is greater than unity,

\[
K = \frac{1 - \left| s_{11} \right|^2 - \left| s_{22} \right|^2 + \left| \Delta \right|^2}{2\left| s_{12}s_{21} \right|} > 1
\]

and the determinant of the S-matrix has a magnitude less than one,

\[
\left| \Delta \right| = \left| s_{11}s_{22} - s_{21}s_{12} \right| < 1
\]

If the twoport satisfies both requirements it is defined to be unconditionally stable. Otherwise it is called potentially unstable (sometimes referred to as potentially stable).

**A new stability definition — the \( \mu \)-factor**

Since the stability definition requires two tests, it is difficult to compare the relative stability of devices. A relatively new development[3] combined the above two tests into a single, more practical form. It is the \( \mu \)-factor, which must be greater than one for stability.

\[
\mu = \frac{1 - \left| s_{22} \right|^2}{\left| s_{11} - \Delta(s_{22}) \right| + \left| s_{21}s_{12} \right|} > 1
\]

The \( \mu \)-factor is very useful to compare the relative stability of devices; the larger it is for a given device, the greater that device’s stability.

Since all of the tests are based on frequency dependent
small signal s-parameters, it is easy to see that twoport stability if a function of frequency. Generally, active devices are stable at the very low frequencies, for which $|s_{12}|$ is very small, and also at the very high frequencies at which $|s_{21}|$ rolls off. Unfortunately for amplifier designers there is a wide range of RF and microwave frequencies for which the possibility of oscillation is a threat to stable operation, as indicated in Figure 2.

**Figure 2.** Broadband stability characterization of a typical microwave transistor showing potential instability in the 20-4000MHz frequency range where $\mu < 1$.

**Stability Circles**

The analytical tests classify the twoport as stable or potentially unstable, and it may also be useful to know what type of terminations could cause problems. Since "a picture is worth a thousand words," we turn to a Smith Chart based graphical technique for elucidation.

A visual illustration of RF stability is available through the use of stability circles[4], whose circumference is the border between the regions of stable and unstable terminations. For each twoport a set of two stability circles can be drawn: one for the source side and one for the load. The centers and radii of the circles are computed from the twoport s parameters, and the circles can be plotted by most of the RF/MW CAE programs. Typical single-frequency output is shown in Figure 3.

Stability circles are computed from the frequency dependents small-signal s-parameters of the device. As a consequence, stability circles also change with frequency.

**Determine the stable side of a source stability circle**

As noted, the circumference of a source stability circle represents the locus of all source terminations that lead to the borderline case between stable and potential unstable output, that is $|\Gamma_{\text{out}}| = 1.0$.

We need to find at least one other source termination that is not on the circumference and investigate whether it causes $|\Gamma_{\text{out}}|$ to be less than unity (stable), or greater than unity (potentially unstable). An obvious choice is 50Ω — the original source used during the initial s-parameter measurements — that resulted in the basic $s_{22}$ of the device. We want to know the magnitude of the output reflection coefficient, when the source is equal to 50Ω.

**Figure 3.** Single-frequency source and load stability circles of a typical RF transistor, indicating potential instability. Since the circles intersect the unity radius circle of the Smith Chart, a portion of the Chart contains terminations that could lead to oscillation.
Then, if \(|s_{22}| < 1.0\) then the 50Ω source is classified as a termination leading to stable output, and if \(|s_{22}| > 1.0\) then the 50Ω source is classified as a termination leading to potentially unstable output.

Once this decision is made, we can label the 50Ω point (center of the Smith Chart) accordingly as stable or potentially unstable, and see if that point is inside or outside of the stability circle. As soon as this additional point is labeled, both sides of the stability circle can be identified[5], (Figure 4). Note that when the circumference of a stability circle crosses the center of the Smith Chart, we have an undefined case, one about which mathematicians might philosophize.

**Case 4a:** \(s_{22} = .50 @ -140^\circ\)  
\(|s_{22}| < 1.0\), and the source stability circle does not enclose the center of the Smith Chart (50 Ω, which is a stable source). Conclusion: outside region stable, inside unstable.

**Case 4b:** \(s_{22} = .66 @ -23^\circ\)  
\(|s_{22}| < 1.0\), and the source stability circle encloses the center of the Smith Chart (which is a stable source). Conclusion: inside stable, outside unstable.

**Case 4c:** \(s_{22} = 1.12 @ 170^\circ\)  
\(|s_{22}| > 1.0\) and the source stability circle does not enclose the center of the Smith Chart (which is now an unstable source). Conclusion: inside stable, outside unstable.

**Case 4d:** \(s_{22} = 1.20 @ -35^\circ\)  
\(|s_{22}| > 1.0\) and the source stability circle encloses the center of the Smith Chart (which is now an unstable source). Conclusion: outside stable, inside unstable.

**Figure 4.** The original \(|s|\) of the device may either be less or greater than unity, resulting in four possible classifications. Unstable sources are indicated by the shaded regions.

**Two possible graphical forms of unconditional stability**

The inside of each stability circle may show the unstable (Cases 4a & 4d) or stable (Cases 4b & 4c) region. For unconditional stability the complete Smith Chart must be declared stable and there are two possibilities:

1) The center and circumference of the stability circle are located outside of the unity radius Smith Chart and the inside of the circle unstable (Figure 5a),
2) The stability circle completely encloses the Chart and the outside of the stability circle shows the unstable region (Figure 5b).

In both cases the complete Smith Chart is in the stable region, so terminations may be chosen freely without any risk of oscillation. Once again, we only show the source circle here and for complete stability analysis we must check both the source and load circles. If both ports are stable, then the device is classified unconditionally stable.

**Caution about multi-stage systems**

In the case of multistage amplifiers, the overall twoport stability factor indicates only whether oscillation is possible at the input or the output of the complete amplifier. Unless every stage is unconditionally stable, oscillation may develop between stages. In such a case, stability can be determined only by examining each interstage loop to see if the conditions for oscillation exist[6].

**Stabilizing an active twoport**

Potentially unstable devices can always be stabilized by an appropriate cascade resistor, an approach that is simple and effective. However, adding a dissipative element throws away transducer gain; and depending whether the resistor applied to the input or output, also sacrifices noise performance or output power. The minimum loss resistance refers to the resistor value that leads to a borderline stability, at which μ just takes on a unity value. Generally speaking, a higher level of stability can be achieved by adding more loss to the device.

Depending on the input-output phase relationship of the device, resistive feedback could also help, and it may be preferable to the brute-force cascade resistive approach. Application of lossless feedback may also improve stability and at the same time control other parameters, such as the optimum noise source reflection coefficient and conjugate input match[7].

**Graphical forms of potential instability**

A potentially unstable situation occurs when the stability circles intersect or are placed entirely inside the Smith Chart. Another form of instability occurs when the stability circle completely encloses the Smith Chart and the inside of the circle represents the unstable region. The three possible unstable combinations are shown in Figure 6. Potential instability does not mean that the twoport oscillates; rather it merely indicates the prospect of negative resistance at one of the ports, which, in turn, may lead to oscillation for some given termination.

**Figure 5.** Two forms of stability for source circles. The shaded areas show the region of unstable sources. In both illustrations the unity radius circle of the Smith Chart is in the stable region. Stability circles are defined by two parameters: the radius, rs, and the center vector, Cc, computed from the s parameters of the twoport.

**Figure 6.** Three possible forms of potentially unstable source terminations. All plots refer to the case for which the inside of the stability circles show the unstable region. Clearly, in all three plots, the shaded parts of the Smith Charts refer to terminations that may lead to oscillation, the right side figure being the worst.
Finding the minimum loss resistor at the input of the device

The minimum loss cascade stabilizing resistor value can be determined easily from the Smith Chart by finding the constant resistance or constant conductance circle that is tangent to the appropriate stability circle. To illustrate the process, we show the two possible choices of stabilization for the device whose source stability circles are previously shown in Case 4a of Figure 4. For this device, the inside region of the source stability circle indicates the unstable region. The constant resistance and constant conductance circles that are tangent to the stable side of the stability circle indicate the minimum loss series resistance and parallel conductance, indicated in Figure 7.

In the above illustration, stabilization was applied at the input of the device, and depending on the signal and noise level of the amplifier, it may be better to stabilize at the output. Sometimes splitting the loss between the input and output leads to the best system performance. Adding the appropriate amount of minimum loss to the input or output stabilizes both sides of the device.

Occasionally, depending on the locations of the stability circles, series or parallel resistive stabilization is not available. In such case, the corresponding tangent constant resistance or constant conductance circle cannot be drawn at the stable side of the stability circle. For example, Case 4d of Figure 4 illustrates a device for which only parallel resistance (conductance) helps. Generally speaking, if the open circuit point of the Smith Chart is unstable then series stabilization is not available; for unstable short circuit points, parallel resistance does not help.

Broadband considerations

We noted that the safest way is to stabilize the device for all frequency, even outside the passband of interest. Adding a stabilizing resistor degrades the performance at all frequencies, and in many cases a frequency selective stabilizing network may be the better choice. Simple RL or RC combinations may sacrifice performance only where it is necessary to improve stability, without effecting other frequencies at which the device already may be stable.

Figure 8 shows the input and output stability circle locations of a typical RF small-signal transistor before and after stabilization, through a broad range of frequencies.
Initially the transistor was potentially unstable between 100MHz and 1300MHz. Unconditional stability was achieved by cascading a series combination of a 110ohm resistor and 40nH inductor across the output of the device.

8a. Potentially unstable device

8b. Device with stabilizing network.

Figure 8. Broadband stability circles plotted from 100MHz to 2000MHz, indicating potential instability between the frequencies of 100-1300MHz (8a). After stabilization, the circles are completely outside of the Chart, (8b), with their outside regions being stable, indicating unconditional stability.

A thorough stability analysis should always be the first step prior to designing active circuits. However, unconditional amplifier stability, although desirable, may not always be practical, since resistive stabilization always reduces performance. Nevertheless, knowledge of the region of terminations that could lead to oscillation is always valuable information to the designer.

References